A REVIEW OF THE USE OF COMPUTER ALGEBRA SYSTEMS IN VIRTUAL LEARNING ENVIRONMENTS

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Abstract

This contribution will present a review on the availability and use of *computer algebra systems* (CASs) in *virtual learning environments* (VLEs) used for teaching mathematics within the context of higher education.

The integration and compatibility of different CASs in VLEs such as the free open-source Moodle environment and their potential in the support of Mathematics education will also be presented.

Keywords: Computer Algebra Systems, Virtual Learning Environment, Learning Management System, Moodle, Mathematics Education

Background

A ***virtual learning environment*** (VLE), also known as a learning management system, is a web-based platform that allows the management of most aspects of a course such as cohort organisation and enrolment, staff resourcing, the development of learning activities and online assessment, the collection of participation statistics, etc. (Paulsen, Nipper, & Holmberg, 2003).

The tools and features that comprise the VLE aim to facilitate a complete learning and teaching experience and include (O'Leary & Ramsden, 2002):

- Communication between tutors and students, e.g., email, discussion boards and virtual chat facilities that support various types of communication: synchronous and asynchronous, one-to-one, one-to-many and many-to-many.
- Self-assessment and summative assessment, e.g., multiple-choice assessment with automated marking and immediate feedback.
- Delivery of learning resources and materials, e.g., through the provision of learning and teaching materials, images and video clips, links to other web resources, online discussion and assessment activities.
- Shared work group areas-- allowing designated groups of students to upload and share files as well as communicate with each other.
- Support for students-- could take the form of communication with tutors or other students and provision of supporting materials such as course information and Frequently Asked Questions (FAQs).
- Student tools, e.g., individual student web pages, *drop boxes* for the upload of course-work, electronic diaries and calendars.

- Management and tracking of students, e.g., usernames and passwords to ensure that only registered students can access the course and analysis of assessment undertaken by students on their use of materials within the VLE.
- Consistent and customisable look and feel --a standard user interface that is easy for students to understand and use. Courses can be individualised with colours, graphics and logos – but the essential mode of use remains constant.
- Structured delivery of information supported by a standard navigation toolbar. Most VLE software assumes that students will work their way through linear sequences of instructional material. Others are more flexible and will accommodate alternative information structures, e.g., multi-path case studies.

There are many ways of using VLEs, ranging from simple uses of a limited range of tools to support face-to-face courses, through to entirely online courses that make sophisticated use of a wide range of the VLE's facilities.

As with any technology used in teaching and learning, VLEs have no intrinsic educational value in themselves. The way in which online courses and online activities are designed and delivered can add value and increase effectiveness. Below are some commonly perceived advantages and disadvantages of using a VLE.

Advantages

1. Easy online delivery of materials.
2. Easy to use for both students and lecturers.
3. Widens student access on and off campus to learning materials and resources.
4. Offers flexible support for educators who do not need to be in a fixed time or place to support and communicate with students.
5. Has the potential for new ways of learning and teaching, such as active and independent learning, which make use of online communication, online assessment and collaborative learning.

Disadvantages

1. Can become a ‘dumping ground’ for materials not designed to be delivered online.
2. Copyright and IPR of materials need to be considered.
3. Off campus access to hardware and networks can be problematic for both students and educators and raises issues of equality. Disability legislation and accessibility to online materials also need to be considered.
4. Need to plan online support carefully to avoid overload.
5. Such independent learning still needs to be guided and supported. Appropriate training and ongoing support is still needed for both students and educators.

Computer Algebra Systems

A ***computer algebra system*** (CAS) is a software package that performs calculation of mathematical problems in a manner that replicates the symbolic approach of a scientist or mathematician rather than using raw numerical computation. Originally developed as a scientific tool for research purposes, these computer programs are not only used for the computation of scientific problems, but also as teaching tools, mostly in science and technology subjects.

Development of CASs started in the 1960s as research tools mainly in the fields of computer science and theoretical physics. Availability was restricted to a handful of universities. The first popular computer algebra systems were muMATH, Reduce, Derive (based on muMATH), and Macsyma.

Currently, there are a number of systems available, which range from open-source to proprietary licensed software and from small user bases to large-scale adoption in industry and Academia. As of today, the most popular commercial systems are Mathematica and Maple, which are commonly used by research mathematicians, scientists, and engineers. On the open source or free-software front, packages such as Sage and SymPy are available.

The use of computer algebra systems has become increasingly important and widespread in mathematics research and teaching. Lavicza (2006), reported a questionnaire study enquiring about mathematicians' use of CAS in mathematics teaching in three countries: the United Kingdom, the United States, and Hungary. The study examined the extent of CAS use in universities, described some CAS-assisted teaching practices of mathematicians, suggested factors that influence technology integration into university level teaching, and highlighted mathematicians' views on the role of CAS in mathematical literacy. The study highlighted the importance of collaboration between mathematicians and educational researchers to enhance technology in mathematics teaching and learning. Responses from 67% of participants in Lavicza's study indicate that they used CAS for their own research at least on an occasional basis (see Table 1).

Table 1

Mathematicians' Use of CAS in Research and Teaching

Frequency		**Never (%)**			**Occasionally (%)**			**Frequently (%)**		
Country	n	HU	US	UK	HU	US	UK	HU	US	UK
CAS in research	1089	33.2	33.9	32.9	34.4	34.4	34.5	32.3	31.7	32.1
CAS in teaching	920	42.3	41.7	53.4	42.3	41	37.6	15.3	17.2	9.1

Note: Adapted from "The Examination of Computer Algebra Systems Integration into University-level Mathematics Teaching" by Z. Lavicza (2006) *Proceedings of the ICMI 17 study conference.*

Lavicza (2006) reported a large number of mathematicians who have acquired strong working knowledge of at least one mathematical software, and this knowledge can be readily utilized for CAS-assisted teaching. Proficiency in the use of a software package offers an advantage to mathematicians over teachers, as they often don't require initial training for software before beginning to use it in their teaching.

Integrating CAS in VLEs

The focus of this study is to explore if it is possible to embed a CAS in a virtual learning environment. Specifically, Moodle VLE will be assessed since this is the VLE currently in use at the author's teaching institution.

Moodle is an open-source virtual learning environment, which has been developed over several years with an emphasis on education. Moodle is claimed to be the world's most widely used school learning management system and is maintained regularly. Moodle's main characteristics are:

- The Moodle VLE is open source, which means no contracts or per user licensing costs to pay each year.
- Moodle can integrate course management structure, performing tasks such as administration and registration.
- Moodle supports personalised learning, reaching outside the teaching centres and better involving tutors, students and members alike.

Moodle is web based, and it is also highly configurable and extensible, allowing it to be customised to meet specific institutional needs. The integration of multimedia elements is typically done at a low level using JavaScript. Integrating a CAS in Moodle will then depend on its ability to merge with typical web-oriented schemas, such as HTML, Java, JavaScript, etc.

Table 2 shows a sample of free and commercial CASs that have been appraised on their ability to be integrated in Moodle.

Table 2

General-purpose Computer Algebra Systems Moodle Integration Capabilities

System	Creator	Integration with VLE
Axiom	Richard Jenks	No
Calcinator	George J. Paulos	Possible, browser based
FxSolver	Equanalysis UG	Possible via JavaScript
Mathics	Jan Pöschko	Possible via Python
Mathematica	Wolfram Research	Yes via .cdf player
Maxima	Bill Schelter et al.	No
OpenAxiom	Gabriel Dos Reis	No
Sagemath	William A. Stein	Possible, browser based
SMath Studio	A. Ivashov	No
SymPy	Ondřej Čertík	Possible, browser based
Yacas	Ayal Pinkus et al.	No

From all systems surveyed, only Mathematica (Mathematica, 1991) has been found to be capable of direct integration with Moodle. Mathematica is a commercial package that has been in development since 1988, and the current iteration is widely used in academic and commercial research. The system itself was developed as a research tool, originally in the field of symbolic mathematics, but over the years it has evolved into a whole system that allows numerical calculation, natural language processing, advanced simulation, etc. It also has a large repository of built-in curated scientific data that is accessible from the CAS itself. This repository is currently used by *artificial assistants* such as Apple's Siri, allowing them to respond to specific questions of any nature.

From a teacher's perspective, all the power of a CAS is no use if its use requires a learning curve that is above the intended learning needs for a particular topic or its use is restricted via license.

To address this Wolfram developed the ***computable document format*** (CDF) -- an electronic document format designed to allow authoring of dynamically generated interactive content using Mathematica (Wolfram, 2015). CDF is a published public format that supports *graphical user interface* (GUI) elements such as sliders, menus and buttons. Content is updated using embedded computation in response to GUI interaction. Contents can include formatted text, tables, images, sounds and animations. CDF supports Mathematica typesetting and technical notation.

The main disadvantage of the CDF is that its authoring requires the use of a licensed version of Mathematica. This disadvantage is reduced by the availability of a large repository (more than 10,000 documents) of free-to-use CDF files accessible at the Wolfram Demonstrations Project.

The Wolfram Demonstrations Project (http://demonstrations.wolfram.com/) is an open-code repository of dynamic computation files that apply to fields such as science, technology, mathematics, art, finance, etc. It is created by Mathematica users from around the world who participate by contributing code and demonstrations that can be immediately accessed via the Wolfram CDF Player plug-in, which is downloadable free of charge.

It is possible to embed demonstrations on Moodle or any other web-based service by copying and pasting a snippet of JavaScript code from the Share section of the demonstration page. Anyone with a CDF Player installed will be able to interact with the demonstration. See Figure 1 for an example.

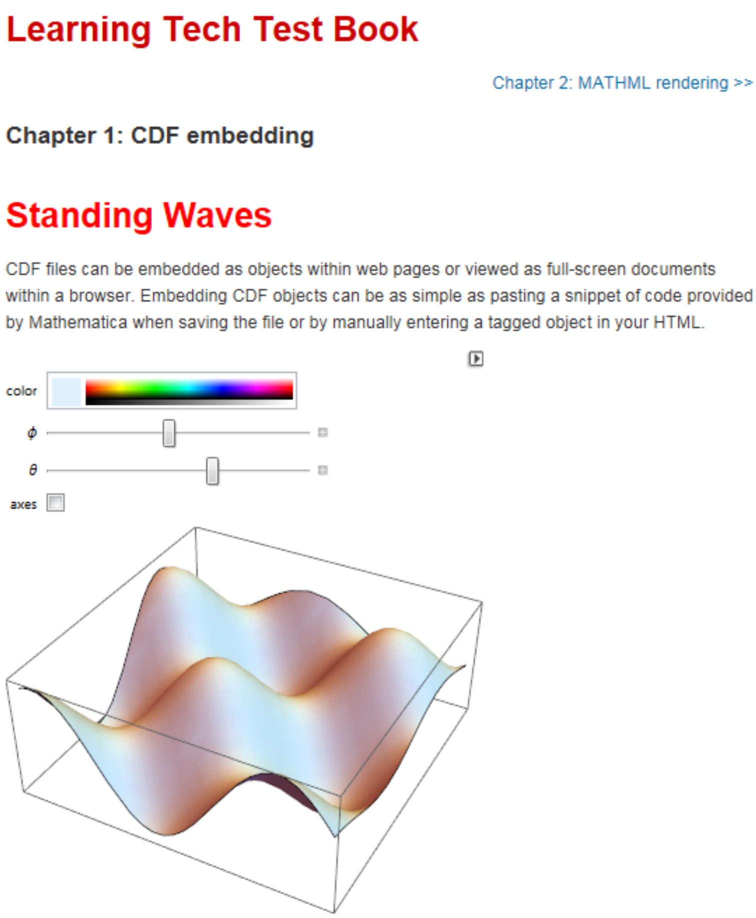

Figure 1. - Integration of a simple interactive CDF in the author's test Moodle environment.

A prototype embedded demonstration was successfully integrated by the author on Moodle showcasing interactive sliders which change the calculation parameters under the hood and update the visualisation instantly. Figure 1 shows a Moodle test page with the embedded CDF as coded by the author.

Two interactive sliders allow one to change parameters of the three-dimensional representation of two multiplying sine waves. The user can rotate the 3-D representation, and all calculations are done in real-time.

Conclusions

The use of CAS as a teaching and learning tool has been reported in several studies, but only on a computer laboratory setting.

The integration of computer algebra systems into virtual learning environments has been explored. Generally, it was found that for the majority of available CASs integration may be possible but would require significant web technology programming and understanding, which is not the typical remit of an educator.

The only CAS that was found to integrate directly with Moodle was Wolfram Mathematica, which is exclusively available commercially. However, the free availability of the CDF Player plug-in and large collection of ready-made demonstrations at the Wolfram Demonstration Project means that there is no need to acquire a license to embed existing interactive scientific demonstrations.

The embedding of computable demonstrations in VLEs is an additional tool to enhance student engagement and understanding that should be explored further.

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