A BLENDED MODEL FOR NON-TRADITIONAL TEACHING AND LEARNING OF MATHEMATICS

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Abstract

This paper suggests a model for teaching and learning mathematics which is based on mathematical phenomenon and on three blended means: technological pedagogical models, representations of mathematical content, and learning contexts. This model moves the student towards deep mathematical learning because of the rich mathematical representations and technological pedagogical tools. Students are motivated to learn mathematics because of the diverse contexts in which they learn (formal, authentic, historical, and virtual) and which add fun to mathematics learning. In the paper, an example is given on how to realize the model in preparing a teaching unit on the golden ratio.

Introduction

This paper treats how to utilize non-traditional methods of teaching and learning mathematics, which combine technology, representations of and connections between mathematical contents, and different learning contexts. At the beginning, a review of literature will be described; one that treats issues related to the use of non-traditional methods and tools for teaching and learning mathematics, and then one that is related to instructional models for non-traditional learning and teaching of mathematics.

Non-Traditional Methods and Tools to Teach and Learn Mathematics

Researchers suggest non-traditional teaching and learning methods to be used in the mathematics classroom. This suggestion comes to motivate and encourage the students to learn mathematics and, from the other side, to facilitate their deep learning and reflection about their learning (Bradley, Notar, Herring, & Eady, 2008; Ufuktepe & Ozel, 2002). This motivation, encouragement and facilitation is supposed to increase their achievement (Boekaerts, 2002), in our case in mathematics.

Helman and Horswill (2002) applied five non-traditional teaching techniques to an introductory statistics course for psychology undergraduates: active and implicit learning, use of breaks, mastery learning, peer tutoring, and problem-based learning. The results demonstrated that undergraduate psychology students who participated in the course outperformed a matched group of students who completed a previous course without these non-traditional teaching techniques.

Several researchers reported the effectiveness of non-traditional methods to teach and learn mathematics. These non-traditional methods involve learning in different settings (for example, out of class); learning in various groupings of students (individually, in pairs, in groups); learning with different types of texts (verbal, audio, video, web, software, etc.); and learning with different tools (physical manipulatives and virtual manipulatives). In this article we want to describe non-traditional teaching methods which involve different settings, groupings, texts and tools. Below we describe research done on using electronic tools in mathematics education, where the learning occurred, in some cases, out-of-class.

Regarding the use of applets in mathematics education, Daher (2009a) found that pre-service teachers who used applets to solve mathematical problems “emphasized the role of applets as fostering, facilitating and clarifying mathematical problems' statement and solving.” The author also pointed that the pre-service teachers looked at applets as tools which learners enjoy working with, so they will look to solving mathematical problems using them. Moyer, Niezgoda and Stanley (2005) tried applets as a non-traditional teaching tool and found that students develop more complex understandings of mathematical concepts when using the applets.

Another non-traditional tool which is used to teach and learn mathematics is the mobile phone. Regarding this use, Baya’a and Daher (2009), for example, found that middle school students perceive learning with mobile phones as one which enabled (1) exploring mathematics independently, (2) learning mathematics through collaboration and team work; where the collaboration is based on equal terms, (3) learning mathematics in a societal and humanistic environment, (4) learning mathematics in authentic real life situations, (5) visualizing mathematics and investigating it dynamically, (6) carrying out diversified mathematical actions using new and advanced technologies, and (7) learning mathematics easily and efficiently. Baya’a and Daher (2010) found that students, learning out of class, using the mobile phone, controlled their learning, related mathematics to real life and thus looked at mathematics as an applied science, and worked as mathematicians.

A third new tool which is used in the teaching and learning of mathematics is the spreadsheets. Regarding this use, Sutherland and Rojano (1993, p. 353), for example, describe how the spreadsheets environment supported students to move from “thinking with the specific to the general both in terms of the unknown and the mathematical relationships expressed in the problem." Daher (2009b) found that the spreadsheets simulation was full of implications and ideas how to grasp a mathematical subject which is usually difficult to grasp by students, namely the probability topic. The author pointed that "the spreadsheets were a trusted working environment that the students returned to in order to verify, investigate or explain a mathematical relation or phenomenon" (p. 291).

So, various technological tools were used as non-traditional tools for teaching and learning mathematics, where they were used sometimes in a new setting — the out-of-class setting. In this article we will talk about all of these tools in a broader setting which emphasizes, first of all and mainly, mathematical phenomena as the heart of mathematics learning which utilizes technology.

Instructional Models for Non-Traditional Learning and Teaching of Mathematics

Truxaw and DeFranco (2005) describe three models for teaching mathematics that can be distinguished depending on the discourse continuum, from univocal to dialogic — that is, *deductive* (associated with univocal), *inductive* (associated with dialogic), and *in/deductive*. The deductive model is a top-down approach and involves reviewing concepts and procedures related to the subject matter. The teacher here leads the students, maintaining control and conveying the information via verbal exchanges. The inductive model involves moving from specific cases toward more general hypotheses. The students here are in control of their learning. The in/deductive model begins as in the inductive model but ends as in the deductive model. Our model follows more the in/deductive instruction, where the students investigate independently mathematical concepts and procedures, analyze verbally with their teacher their results, and agree together on the new acquired knowledge.

Wigley (1992) suggested a model for teaching mathematics, where the learner is engaged by fostering a conjecturing atmosphere. This model has the following features: (1) the teacher presents a challenging context or problem to the students who have the time required to work on the problem, so that they can make conjectures about solutions or results, (2) the students' work yields a variety of ways and strategies which help to deal with the problem situation, (3) the strategies are applied to a variety of problems, (4) a variety of techniques is used to help students to review their work, and to identify what they have learned, how it connects together and how it relates to other knowledge. Our model starts too with a challenging problem related to a mathematical phenomenon, where the students work with a variety of technological pedagogical models, use a variety of geometric and algebraic techniques, and make conjectures so to arrive at solutions and results.

Educational researchers have paid attention in the past decade to the importance of suggesting instructional models that suggest how non-traditional learning and teaching should be realized. For example, Mishra and Koehler (2006) emphasize that it is not sufficient for teachers to be knowledgeable about technology or pedagogy in order to use technology efficiently in the classroom. The authors argue that teachers need also to know how technology can be integrated with specific content in meaningful ways. In this article we will suggest a model which takes into consideration the technological content knowledge of teachers which are emphasized by Mishra and Koehler. This model is based on different electronic tools: web tools, dynamic simulations, mobile phones, in addition to other special features which make it context based and content based, and thus the model avoids the problematic aspect of technology based learning and teaching, mentioned by Harris, Mishra, and Koehler (2009) and described below.

Purpose and Rationale

Harris et al. (2009) criticize the techno centricity of different approaches to technology integration in teaching, describing those approaches as often neglecting the content and specificity of the educational field and thus not sufficiently considering the dynamic and complex relationships among content, technology, pedagogy, and context. We intend in this article to suggest a model of learning and teaching which tries to avoid over emphasis on technology as an end for itself. Instead, our model emphasizes the mathematical content by suggesting starting from a mathematical phenomenon, taking care of the mathematical concepts and procedures associated with the phenomenon, choosing, for the concepts and procedures, various mathematical representations and connections, selecting technological pedagogical models which fit the mathematical phenomenon and concepts, and building, using the technological pedagogical models, mathematical activities for varied learning settings. This model would shift the emphasis from the techno-aspect of the learning to its experiential aspect.

The model is part of a longitudinal study that began this academic year. We built the theoretical model based on our experiences as mathematics teachers, as lecturers of mathematics pedagogy and educational technology courses, as pedagogical mentors for pre-service teachers in a teacher training college, and as researchers in the field of ICT in education generally and in mathematics teaching specifically. We are applying the model with our pre-service teachers who build using it mathematics teaching units which they will utilize in teaching middle school students mathematics. We intend to study the effect of the construction and use of these units on the teaching/learning processes, attitudes of students toward learning mathematics, students' motivation for learning mathematics, and students' perception of mathematics. This article will describe the model arrived at in the first phase of the study.

The Blended Model

This model has four characteristics:

(1) It suggests mathematical phenomena as the heart of learning mathematics. Phenomena could come from real life, authentic situations, history, literature, etc.

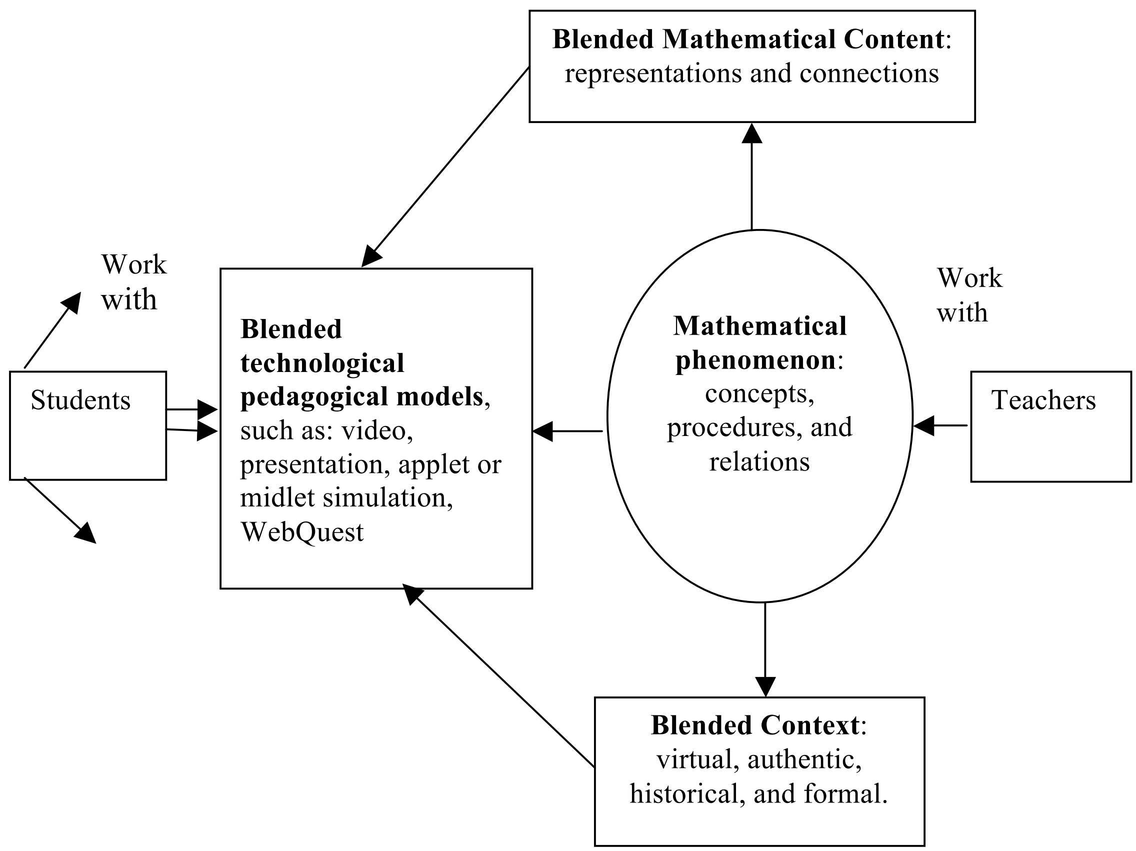
(2) It depends on using various technological pedagogical models for learning mathematics, for example: Dynamic simulations (built using applets, midlets or spreadsheets), WebQuest activities, electronic presentations, videos, etc.

(3) It suggests blended mathematics which is mathematics composed of different representations (algebraic, geometric, verbal, etc.)

(4) It suggests learning which occurs in various contexts: virtual, authentic, historical, and formal.

Figure 1 represents our suggested model.

Figure 1: A blended model for non-traditional teaching and learning of mathematics



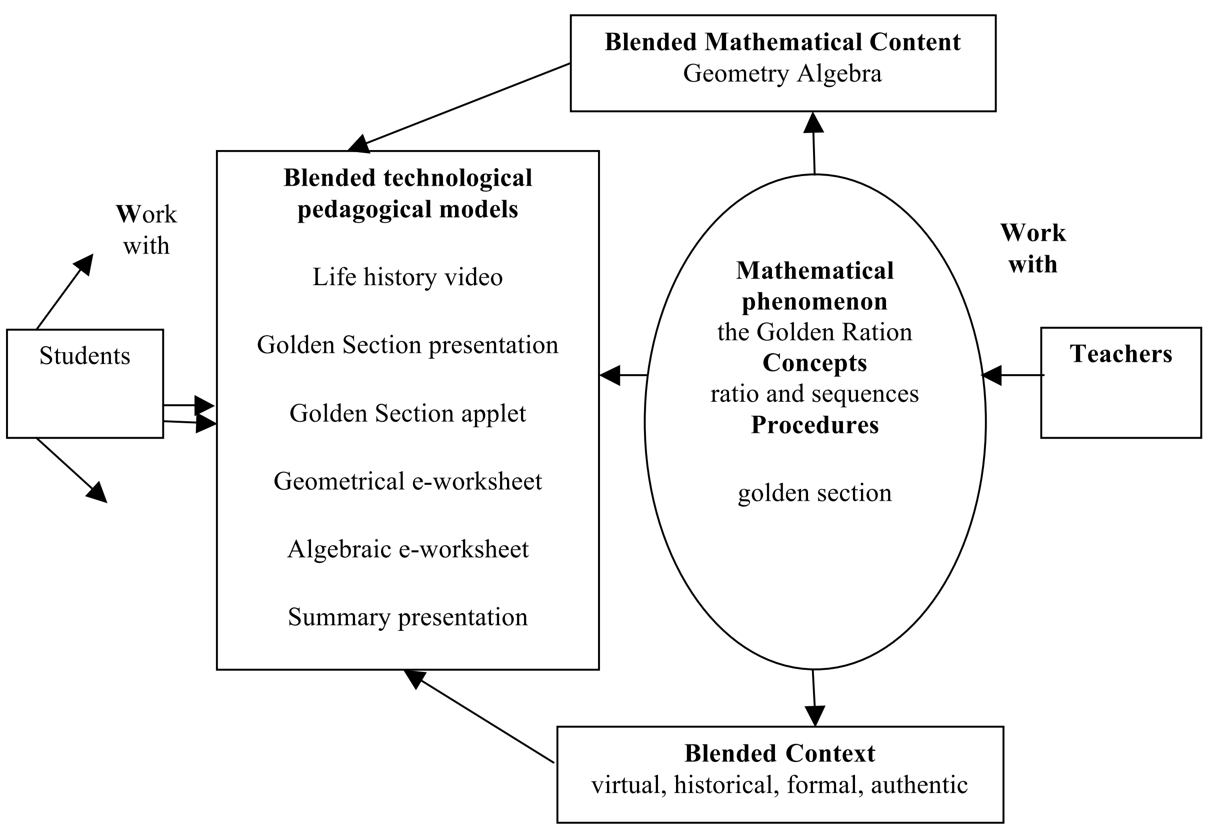
Description of Teaching and Learning Actions According to the Blended Model

The teacher starts from the mathematical phenomenon. Here, the mathematical phenomenon relates to the mathematical concepts, procedures and relations that the teacher wishes to teach. Then, the teacher decides regarding two issues: which mathematical representations he/she will involve the students with and in which context the students will learn the mathematical concepts, procedures and relations. For example, if the teacher wants to teach the subject of the circle area, he/she can choose to teach the subject formally without integrating real life or history. Alternatively, he/she can choose to teach the subject starting from the historical Greek method of finding the area of the circle and then relating that method to the current method of finding the area of the circle. Investigating the Greek historical method, the teacher lets the students work with geometric and algebraic methods to find the area of the circle. The teacher then decides regarding the technological pedagogical models that he/she will use to prepare the learning materials. This decision influences the role of the student. For example deciding to present one of the historical methods via a video or a presentation, the student's role will be relatively passive. On the other hand, deciding to let the student experiment with an applet to find the area of the circle following the historical method gives the students more active role. Thus, the student's role is influenced directly from the technological pedagogical models that the teacher chooses to work with and/or to let the students work with. This role is also influenced, but indirectly, from the context in which the teacher chooses to let the students work. For example, working in an authentic context gives the student more active role than working in a formal context. In addition the blending of technological pedagogical models, learning context and mathematical content would make the students go through various learning experiences and thus enrich and deepen their learning.

An Example

We show first in Figure 2 the example as it fits in Figure 1 above and then describe how a pre-service teacher worked to prepare a teaching unit according to the model.

Figure 2: Planning the golden ratio unit according to the model



The pre-service teacher decided to prepare a teaching unit on the golden ratio phenomenon according to the model. She determined first the mathematical concepts and procedures associated with the phenomenon: ratio, sequences, and the golden section. The pre-service teacher then turned to decide on two issues: which mathematical representations suit more the discovery of the golden ratio phenomenon by students and which learning context is essential and/or suits this discovery more than other contexts. After consulting different sources the pre-service teacher decided that geometry methods suit best the discovery of the golden section procedure which was performed by Euclid, and that algebraic techniques can be used to find the value of the golden ratio. After deciding regarding the mathematical representations, the pre-service teacher came to decide regarding the learning contexts appropriate for the phenomenon. The golden ratio phenomenon had four aspects:

- a historical aspect (because it is related to Pythagoras and Euclid);
- a real life out-of-class authentic aspect (because it is related to real life phenomena like ratios in the human body, in buildings, in drawings, etc.);
- a virtual aspect (because it is associated with phenomena that could be researched virtually, for example using Google Earth); and
- a formal aspect (because some formal mathematical equations and rules are associated with the golden ratio phenomenon).

So, the pre-service teacher decided to prepare learning materials for the students that make them work in the four contexts. The pre-service teacher then decided upon the order of working in the four contexts: historical context, formal context, real life authentic context, and virtual context.

Time came to decide upon the technological pedagogical models that would be used in the unit. Here the issue to consider was: which technological pedagogical model suits best (1) presenting the life histories and mathematical work of Pythagoras and Euclid, (2) discovering the mathematical concepts or procedures associated with the phenomenon, (3) researching real life phenomena out-of-class and relating it to the golden ratio phenomenon, and (4) researching real life phenomena or other phenomena virtually. Table 1 describes which technological pedagogical models the pre-service teacher chose for each context.

Table 1: The technological pedagogical models used by a pre-service teacher to prepare a mathematical teaching unit on the golden ratio topic

Context	**Technological pedagogical models**
Historical context	• **Life history video:** to present the life history and contributions of Pythagoras and Euclid. • **Mecca video:** to present proportions associated with the location of Mecca and related to the golden ratio.
Formal context	• **Electronic presentatio**n: to present the golden section procedure including the subdivision (division of a segment into equal extreme and mean ratios) done by the Pythagoreans. • **Applet**: to help the students notice the self-propagation property of the golden section on a given segment. • **Electronic worksheet**: to further help the students find geometrically the location of the point that satisfies the conditions of the golden section as did Euclid, and to let the students prove geometrically the correctness of the procedure • **Electronic worksheet**: to help the students find algebraically the value of the golden ratio resulting from the golden section. • **Spreadsheet simulation**: to learn about the Fibonacci sequence and associate it with the golden ratio. • **Electronic presentation**: to summarize the conclusions and results that the students were supposed to arrive at.
Authentic context	• **Electronic presentation**: to present directions about working with the midlet Fit2Go (Yerushalmy & Weizman, 2007). • **Mobile phone authentic activities**: To investigate the golden ratio in real life phenomena (for example, the ratio between height of a person and the height of his navel, or the ratios of length and width of doors and windows in buildings). The activities consisted of field tasks that could be solved using mobile phone and a midlet called Fit2Go that enables students to assign points which fit real life measures (for example, height of a person and height of navel) and to draw a function which fits the points, if possible.

Virtual context	• **Electronic presentation**: to present directions about working with a WebQuest activity. • **Electronic presentation**: to present directions about working with Google Earth. • **WebQuest activity:** to make the students expand their knowledge regarding the realization of the golden ratio phenomenon in real life. The WebQuest sites contained videos, pictures and drawings of various figures that include proportions equal to the golden ratio. • **Google Earth activity:** to let the students calculate the proportion of the distance between Mecca and the north pole to the distance between Mecca and the south pole (which is equal to the golden ratio).

The lesson plans of the golden ratio unit (three-hour unit) are:

<u>First hour</u>
Introduction: Presenting the video of the life history and mathematical work of Pythagoras and Euclid, presenting the golden section procedure using an electronic presentation.

Investigation and discovery: The students here investigate the golden ratio (1) dynamically using an applet, (2) geometrically using an electronic worksheet, (3) algebraically using also an electronic worksheet, and (4) dynamically using a spreadsheet.

Summary: Summarizing the work, results and conclusions expected to be found by students using an electronic presentation.

<u>Second hour</u>
Introduction: Using electronic presentations to present directions about working with WebQuest activities, using electronic presentations to present directions about working with Google Earth.

Investigation and discovery: The students first work with a WebQuest to expand their knowledge of the golden ratio phenomenon, and then they watch a video on measurements associated with Mecca and the golden ratio. Afterwards they work with Google earth to measure the distance of Mecca from the North and the South Poles.

Summary: Using an electronic presentation to summarize the work, results and conclusions expected to be found by students as a result of working with the WebQuest and with Google Earth.

Third hour
Introduction: Using an electronic presentation to present directions about working with the midlet Fit2Go.

Investigation and discovery: The students work out-of-class with the mobile phone and the midlet Fit2Go to investigate real life phenomena. The students also use the mobile phone to document their work and observations.

Summary: Using an electronic presentation to summarize the work, results and conclusions expected to be found by students as a result of working with the mobile phone.

Conclusion

This article describes a blended model for non-traditional teaching and learning of mathematics based on researching a mathematical phenomenon. This model utilizes blended representations of mathematical content, blended learning contexts and blended technological pedagogical models.

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